



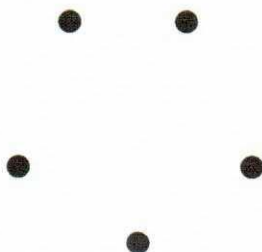
*5<sup>th</sup> Challenge for Future Mathematicians*  
*Bogor, Oktober 27-30, 2018*

Upper Primary School Category  
Team Contest-**SOLUTION**

Time : 80 minutes



1. There are five points as shown in the figure below. Select any two points and connect them to form a straight line. How many different straight lines can we form?



**Answer:**

There are 4 lines between the first point and other four points.

There are 3 lines between the second point and the other three points except the first point.

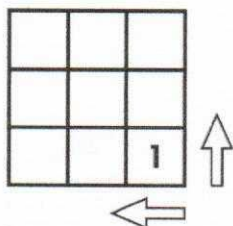
There are 2 lines between the third point and other two points except the first and second points.

There is 1 line between the fourth and fifth points.

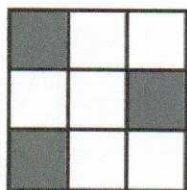
Therefore, there are a total of  $4 + 3 + 2 + 1 = 10$  lines.

2. Each square to decode is filled as follows:

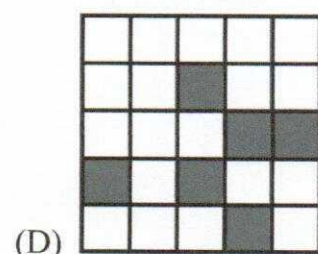
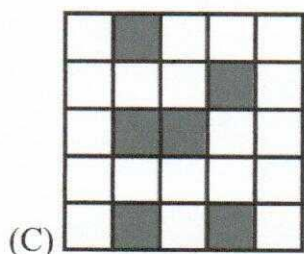
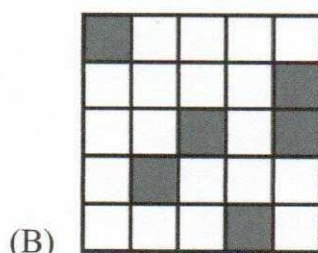
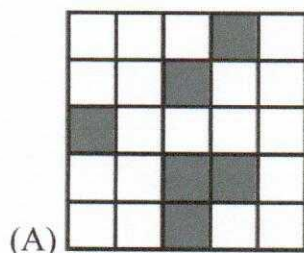
- Each big square contains many small squares.
- Each small square is filled with a number.
- First, the small square at the bottom on the right is filled with number 1.
- Then, from right to left, the next number is 2 times of the previous number.
- Then, from bottom to top, the next number is 2 times of the previous number.



A CFM code of a square is the sum of the numbers of the grey cells. For example, the CFM code of the square below is:  $2 + 16 + 4 = 22$ .



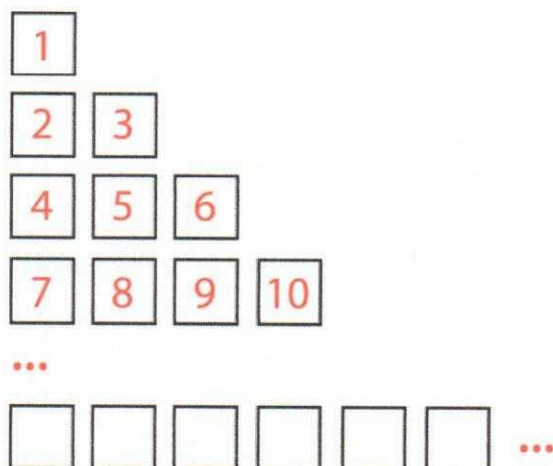
What is the sum of the code below?



**Ans: (B)**

*Hint: code (B) is 302*

3. The numbers below are arranged following a pattern:



Which row and column does the number 1000 belong to?

**Ans:** Row 45<sup>th</sup>, column 10<sup>th</sup>

**Solution:** 40 pt

The 1<sup>st</sup> row has 1 number, the 2<sup>nd</sup> row has 2 numbers,....., so the n<sup>th</sup> row has n numbers.

The first n rows has:

$$1 + 2 + \dots + n = n(n+1)/2 \text{ numbers. (10pt)}$$

So the first 44 rows has 990 numbers. (10pt)

Therefore, the number 1000 belongs to the 45<sup>th</sup> row (10pt) and the 10<sup>th</sup> column (10pt).

4. What is the remainder of the following expression  $\underbrace{111\dots1}_{1000} \div 7$ .

**Answer:**

Because 111111 can be divided by 7 evenly,  $1000 \div 6 = 196\dots4$ , therefore

$$\underbrace{111\dots1}_{1000} \div 7 = \underbrace{111\dots10000}_{996} \div 7 + 1111 \div 7.$$

In the right side, the remainder in the first division is 0, and the remainder of  $1111 \div 7$  is 5.

5. Five participants in a competition get a total of 404 points and their individual scores are different. Among them, the first placer gets 90 points. Find the least and highest possible score of the last placer. (The number of points for each participant is a whole number.)

**Answer:**

The participant in last place gets at least  $404 - (90 + 89 + 88 + 87) = 50$  points.

Four participants except the first placer get  $(404 - 90) \div 4 = 78.5$  points on average, so four numbers close to 78.5 are 77, 78, 79 and 80. Therefore, the last place participant gets at most 77 points.

Proposed Marking Scheme:

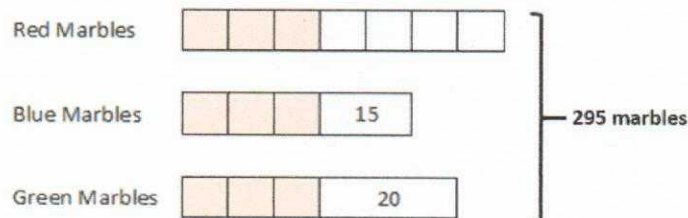
- Get the minimum possible score (up to 20 points)
- Get the maximum possible score (up to 20 points)



6. Mell had a total of 295 red, blue, and green marbles. He gave  $\frac{4}{7}$  of the red marbles, 15 blue marbles, and 20 green marbles to Anna. After that, Mell had an equal number of red, blue, and green marbles. How many Green marbles did she have at first?

**Answer:**

We may describe the number of the marble like the picture below



The red shaded area is the marbles left after Mell gave her marble to Anna.

$$13\square + 15 + 20 = 295$$

$$13\square = 260$$

$$\square = 20$$

So, the number of red marble that Mell have at first =  $3\square + 20 = 3 \times 20 + 20 = 80$  green marbles

Proposed Marking Scheme:

- Set up the equations/logic to get the correct answer (up to 20 points)
- Get the correct answer (20 points)

7. The smallest positive integer  $n$  for which  $n(n+1)(n+2)$  is a multiple of 5 is  $n = 3$ . All positive integers,  $n$ , for which  $n(n+1)(n+2)$  is a multiple of 5 are listed in increasing order. What is the 2018<sup>th</sup> integer in the list?

**Answer:**

$n = 3, 4, 5, 8, 9, 10 \rightarrow$  in a block of 10 integers, 6 are multiples of 5.

Note:  $2018 = 336 \times 6 + 2$ . 336 block of 10 = 3360

This means that, in the first  $336 \times 10 = 3360$  positive integers, there are  $336 \times 6 = 2016$  integers

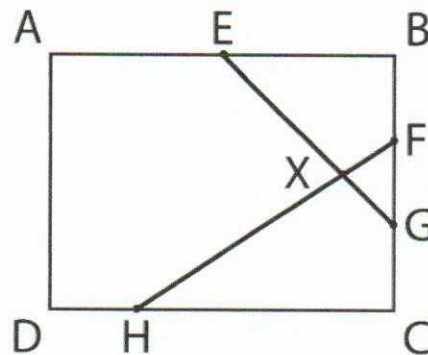
$n$  for which  $n(n+1)(n+2)$  is a multiple of 5.

The 2017<sup>th</sup> integer ends with 3  $\rightarrow 3363$ , the 2018<sup>th</sup> integer ends with 4  $\rightarrow 3364$

Proposed Marking Scheme:

- Observe that there are for every 10 integers, there are 6 integers that are multiples of 5 (5 points)
- Deducing properly the conditions (up to 15 points)
- Deducing to get the correct answer (20 points)

8. Consider a rectangle ABCD. E is the midpoint of AB. F and G are on segment BC so that:  $BF=FG=GC$ . H is on segment DC so that:  $DH = \frac{1}{4} DC$ . EG intersects FH at the point X. What is the ratio of the area of the triangle XFG to the area of the rectangle ABCD?



**Solution:** 40pt:

$$S(EFG) = \frac{1}{12} S(ABCD)$$

$$S(EHG) = S(EBCH) - S(EBG) - S(HGC) = \left(\frac{5}{8} - \frac{1}{6} - \frac{1}{8}\right) S(ABCD) = \frac{1}{3} S(ABCD)$$

$$S(HFG) = \frac{1}{8} S(ABCD)$$

$$\Rightarrow S(EFG)/S(EHG) = \frac{1}{4} = S(XFG)/S(XHG) \quad (20 \text{ points})$$

$$\Rightarrow S(XFG)/S(HGF) = \frac{1}{5} \quad (10 \text{ points})$$

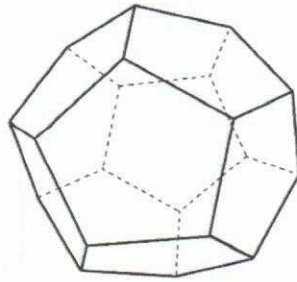
$$\Rightarrow S(XFG) = \frac{1}{40}$$

(10 points)

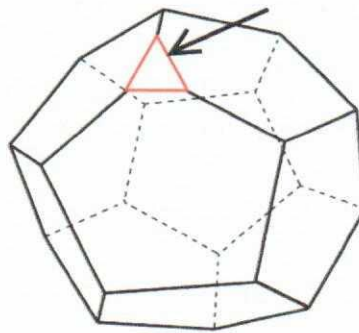
Proposed Marking Scheme:

- Correctly solve all the ratios of the figure (up to 30 points)
- Arrive with the correct answer (10 points)

9. Consider a regular dodecahedron which have 12 similar faces as in the figure below:



We cut each vertex of that dodecahedron by a plane, for example, one vertex is cut as in the figure below. When the cutting is done, how many sides does that dodecahedron have?



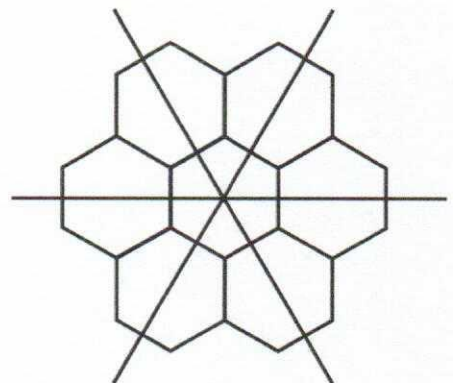
**Ans: 90**

*Hint: The original shape has 20 vertex and 30 edges. For each vertex, we have 3 more edges, so in total, we have:  $30 + 20 \times 3 = 90$  (edges).*

**Proposed Marking Scheme:**

- Observe that there are 20 vertex (10 points)
- Observe that there are 30 edges (10 points)
- Arrive with the correct answer (20 points)

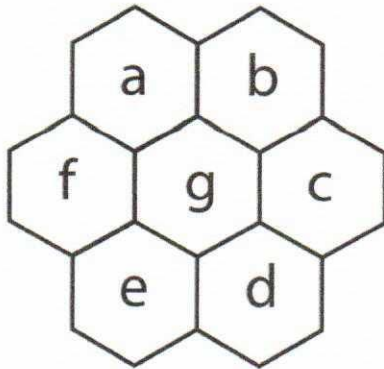
10. In each cell, a number from 10 to 16 is filled, each number is used once. The 3 sums of 3 numbers in the 3 lines are equal. How many ways of filling are there? Given that 2 ways of filling are considered the same if they can be received by rotating.





**Solution: 40pt**

Assume the number in each cell by a letter as in the figure below:



One has that:  $a + d = b + e = c + f$ .

On the other hand,  $a + b + c + d + e + f + g = 10 + \dots + 16 = 91$ , or:

$$3(a+d) + g = 91.$$

So  $g$  is either 10 or 13 or 16. (5 points)

- $g = 10$ : the 3 pairs of numbers on each line can be: (11,16); (12,15); (13,14). (5 points)
- $g = 13$ : the 3 pairs of numbers on each line can be: (10, 16); (11, 15); (12, 14). (5 points)
- $g = 16$ : the 3 pairs of numbers on each line can be: (10, 15); (11, 14); (12, 13). (5 points)

For each value of  $g$ , for the 1<sup>st</sup> pair, we have one way to fill, for the 2<sup>nd</sup> pair, we have 4 ways to fill, for the 3<sup>rd</sup> pair, we have 2 ways to fill. So for each value of  $g$ , we have 8 ways to fill. (10 points)

- So finally, we have 24 ways to fill. (10 points)