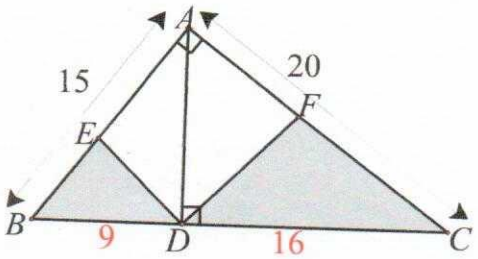
5th Challenge for Future Mathematicians
Bogor, Oktober 27-30, 2018
Junior High School Category
Individual Contest-**SOLUTION**
Time : 90 minutes

PART I.

1	$C > B > M > F$	9	901
2	256	10	$4\sqrt{21}$
3	5	11	43
4	7	12	$\frac{\pi}{64}$
5	25	13	24
6	1	14	CFFM
7	480	15	619737131179
8	65°	16	162 ways

PART II.

1.



AD = 12, BD = 9, DC = 16
AE : EB = 4 : 3
AF : FC = 3 : 4
$\Delta ABD = \frac{1}{2} 9 \times 12 = 54$,

$\Delta BED = 3/7 \times 54 = 162/7$
$\Delta ACD = \frac{1}{2}\ 16 \times 12 = 96,$
$\Delta DFC = 4/7 \times 96 = 384/7$
Answer $= 546/7 = 78$

Proposed Marking Scheme:

- Get the correct lengths of AD, BD and DC **(1 point each, total 3 points)**
- Get the correct ratios of AE: EB and AF: FC **(1 point each, total 2 points)**
- Get the correct area of ΔBED **(2 points)**
- Get the correct area of ΔDFC **(2 points)**
- Get the final answer **(1 point)**

Note: If correct answer only with no solutions, give 1 point.

2. Since GCD $2n + 9$ and $7n + 5$ is T.
then $2n+9=a\cdot T$ and $7n+5=b\cdot T$, which a and b are positive integers and co-prime.
If we do subtract $2n + 9$ from $7n + 5$ then we get:

$$(7n+5)-(2n+9)=b\cdot T-a\cdot T \Rightarrow 5n-4=(b-a)\cdot T \Rightarrow n=\frac{bT-aT+4}{5} \ \ldots \text{(I)}$$

If we add $2n + 9$ to $7n + 5$ then we get:

$$(7n+5)+(2n+9)=b\cdot T+a\cdot T \Rightarrow 9n+14=(b+a)\cdot T \ldots \text{(II)}$$

Substitute value of n from (I) to (II)

$$9n+14=bT+aT$$

$$9\left(\frac{bT-aT+4}{5}\right)+14=bT+AT$$

$$9bT-9aT+36+70=5bT+5aT$$

$$14aT-4bT=106$$

$$T=\frac{106}{14a-4b}=\frac{53}{7a-2b}$$

Since T, a, and b are positive integers so $7a - 2b$ is a factor of 53, then the possible factors for T are 1 and 53, The sum of possible values of T is $1 + 53 =$ **54.**

Proposed Marking Scheme:

- Set-up the general forms of $2n+9$ and $7n+5$ **(1 point each, total of 2 points)**
- Manipulate and get the general form of T **(up to 2 points)**
- Deduce that the only possible value of T is 1 and 53 **(2 points each, total of 4 points)**
- Get the final answer **(2 points)**

Note: If correct answer only with no solution, give 1 point.