Australian Mathematics Trust
www.amt.edu.au

Australian Intermediate Mathematics Olympiad 2016

Questions

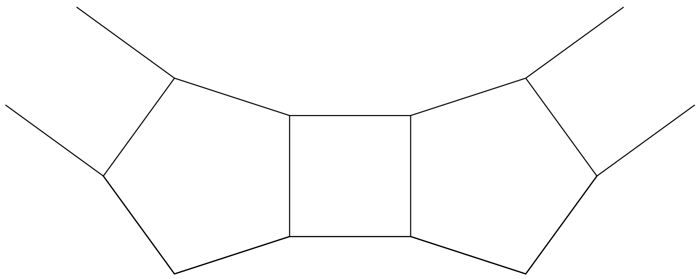
1. Find the smallest positive integer x such that $12x = 25y^2$, where y is a positive integer.

[**2 marks**]

2. A 3-digit number in base 7 is also a 3-digit number when written in base 6, but each digit has increased by 1. What is the largest value which this number can have when written in base 10?

[**2 marks**]

3. A ring of alternating regular pentagons and squares is constructed by continuing this pattern.



How many pentagons will there be in the completed ring?

[**3 marks**]

4. A sequence is formed by the following rules: $s_1 = 1, s_2 = 2$ and $s_{n+2} = s_n^2 + s_{n+1}^2$ for all $n \geq 1$. What is the last digit of the term s_{200}?

[**3 marks**]

5. Sebastien starts with an 11×38 grid of white squares and colours some of them black. In each white square, Sebastien writes down the number of black squares that share an edge with it.

Determine the maximum sum of the numbers that Sebastien could write down.

[**3 marks**]

6. A circle has centre O. A line PQ is tangent to the circle at A with A between P and Q. The line PO is extended to meet the circle at B so that O is between P and B. $\angle APB = x^\circ$ where x is a positive integer. $\angle BAQ = kx^\circ$ where k is a positive integer. What is the maximum value of k?

[**4 marks**]

PLEASE TURN OVER THE PAGE FOR QUESTIONS 7, 8, 9 AND 10

7. Let n be the largest positive integer such that $n^2 + 2016n$ is a perfect square. Determine the remainder when n is divided by 1000.

[**4 marks**]

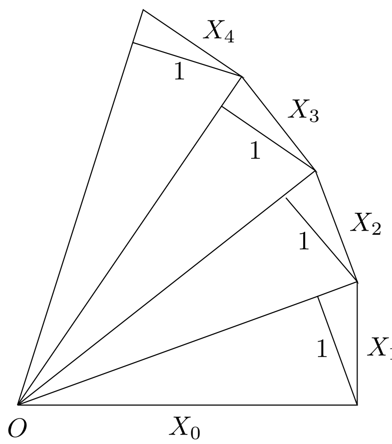
8. Ann and Bob have a large number of sweets which they agree to share according to the following rules. Ann will take one sweet, then Bob will take two sweets and then, taking turns, each person takes one more sweet than what the other person just took. When the number of sweets remaining is less than the number that would be taken on that turn, the last person takes all that are left. To their amazement, when they finish, they each have the same number of sweets.

They decide to do the sharing again, but this time, they first divide the sweets into two equal piles and then they repeat the process above with each pile, Ann going first both times. They still finish with the same number of sweets each.

What is the maximum number of sweets less than 1000 they could have started with?

[**4 marks**]

9. All triangles in the spiral below are right-angled. The spiral is continued anticlockwise.



Prove that $X_0^2 + X_1^2 + X_2^2 + \cdots + X_n^2 = X_0^2 \times X_1^2 \times X_2^2 \times \cdots \times X_n^2$.

[**5 marks**]

10. For $n \geq 3$, consider $2n$ points spaced regularly on a circle with alternate points black and white and a point placed at the centre of the circle.

The points are labelled $-n, -n+1, \ldots, n-1, n$ so that:
(a) the sum of the labels on each diameter through three of the points is a constant s, and
(b) the sum of the labels on each black-white-black triple of consecutive points on the circle is also s.

Show that the label on the central point is 0 and $s = 0$.

[**5 marks**]

Investigation

Show that such a labelling exists if and only if n is even.

[**3 bonus marks**]